

Relations and Functions

Last-minute revision: the key ideas, standard results and traps of every part. The formula sheet is the last page.

If you only have five minutes

- ① Relation = any subset of $A \times B$: 2^{pq} relations.
 Reflexive 2^{n^2-n} , symmetric $2^{(n^2+n)/2}$.
- ② Equivalence relations on n elements: Bell number B_n
 (1, 2, 5, 15, 52).
- ③ Function: every input exactly one output. One-one:
 $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$; onto: range = codomain.
- ④ Onto maps $n \rightarrow m$: $\sum (-1)^{r+m} C_r (m-r)^n$. $\text{gof one-one} \Rightarrow f$ one-one; $\text{gof onto} \Rightarrow g$ onto.
- ⑤ f^{-1} exists $\Leftrightarrow$ bijective; mirror in $y = x$; $(\text{gof})^{-1} = f^{-1} \circ g^{-1}$.
- ⑥ Wavy curve: + far right, change at odd powers, bounce at even; never include denominator zeros.
- ⑦ Range: complete the square, bound the inside; discriminant method, then check the ends.
- ⑧ $a \cos x + b \sin x \in [-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$;
 $\sin^{-1}(\sin x) = x$ only on $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
- ⑨ $[-2.3] = -3$, $\{-2.3\} = 0.7$. Periods: LCM, then test smaller. $f(x+k)$ moves left.

1 Relations: Definition, Domain and Range

- A relation is any subset of $A \times B$; no rule needed.
- Domain: first elements; range: second elements; codomain: all of B .
- R^{-1} flips pairs: $\text{Dom } R = \text{Range } R^{-1}$.
- Identity $I_A = \{(a, a)\}$; reflexive relations may contain more.

STANDARD RESULTS

$$n(A \times B) = pq \cdot \text{relations: } 2^{(pq)} \cdot \text{on } A: 2^{(n^2)}$$

TRAPS

- ✗ **Forgetting 0 when the set is W** Whole numbers start at 0, so (0, 5) and (5, 0) count.
- ✗ **Counting (p, p⁰) twice** $(p, p^0) = (p, 1)$ is in both R_1 and R_2 , so it cancels in $R_1 - R_2$.
- ✗ **Thinking a reflexive relation must be the identity** Reflexive means every (a, a) is in R ; R may contain more pairs.

2 Reflexive, Symmetric and Transitive Relations

- Reflexive: whole diagonal; symmetric: mirror pairs; transitive: every chain's shortcut.
- A chain that returns, (a, b), (b, a), forces (a, a).
- One counter-example disproves a property.

STANDARD RESULTS

$$\text{Reflexive } 2^{(n^2 - n)} \cdot \text{symmetric } 2^{((n^2 + n)/2)} \cdot \text{both } 2^{((n^2 - n)/2)}$$

$$ab = 100a + b \text{ on } \{1, \dots, 130\}: n(R) = 7, \text{ add 6 for symmetry.}$$

TRAPS

- ✗ **Assuming transitivity because no counter-example comes to mind** Test the chain through every pair, including chains that return.
- ✗ **Adding a mirror pair for every pair of R** A pair (a, a) is its own mirror and needs nothing.

✗ **Calling a relation reflexive when one (a, a) is missing** Every element of A needs its diagonal pair.

3 Equivalence Relations and Bell Numbers

- Equivalence = reflexive + symmetric + transitive; it partitions A into classes.
- Classes are identical or disjoint and cover A .
- Equivalence relations on n elements: B_n .

STANDARD RESULTS

$$B_1 \dots B_5 = 1, 2, 5, 15, 52$$

$$\{(1, 2), (2, 1)\} \text{ on } \{1, 2, 3\}: \text{add 4 (reflexive, not symmetric) or 5 (reflexive, symmetric, not transitive).}$$

TRAPS

- ✗ **Counting partitions and forgetting the all-apart one** I_A is an equivalence relation too: it is the partition into singletons.
- ✗ **Adding one pair for symmetry and ignoring the new chains it creates** Every added pair can break or force transitivity; recheck.
- ✗ **Reading $\subset$ as proper subset in the P(X) question** Then R isn't even reflexive; the intended reading is $\subseteq$.

4 Functions: Definition, Intervals and Domain

- Hollow dot: end out; filled: end in. Check each end separately.
- Function: every element of A has exactly one image; domain = A , range $\subseteq$ codomain.
- Largest domain: where the rule makes sense ($\sqrt{x}$ needs $x \geq 0$).
- $x^2 + y^2 = 16$ is not a function but splits into $y = \pm\sqrt{16 - x^2}$.

TRAPS

- ✗ **Getting interval ends wrong in $A - B$** $-1 \notin B$, so it stays in $A - B$ (closed); $2 \in A$, so it leaves $B - A$ (open).

- ✗ **Rejecting a function because two inputs share an output** Only one input with two outputs breaks the definition.
- ✗ **Using the horizontal line to test for a function** The vertical line test decides function or not.

TRAPS

- ✗ **Reversing which function is forced** $\text{gof one-one} \Rightarrow f \text{ one-one}$; $\text{gof onto} \Rightarrow g \text{ onto}$.
- ✗ **Counting onto maps as m^n** Subtract the maps that miss elements (inclusion–exclusion).
- ✗ **Calling a function onto without the codomain** Onto is always onto a stated set.

5 Algebra of Functions and Composite Functions

- $f \pm g, fg$: $D_f \cap D_g$; f/g : also remove $g(x) = 0$.
- $\text{gof}(x) = g(f(x))$ needs $f(x) \in D_g$; usually $\text{gof} \neq \text{fog}$.
- Piecewise fog: graph g , shade the bands for f 's pieces, discard empty cases.

STANDARD RESULTS

Illustration 1.35: fog domain $[-1, \sqrt{2}]$, range $[1, 2] \cup (3, 5]$ (book misprints $[0, 2]$).

TRAPS

- ✗ **Taking the domain of fog as the domain of g** fog also needs $g(x)$ inside D_f ; here $[-1, 3]$ shrinks to $[-1, \sqrt{2}]$.
- ✗ **Keeping cases where the two conditions share no x** $g = x + 2 \leq 1$ needs $x \leq -1$ and $x \geq 2$: drop it.
- ✗ **Assuming fog = gof** $\sqrt{x-2}$ and $\sqrt{x} - 2$ differ in formula and domain.

8 Inverse Functions

- f^{-1} exists $\Leftrightarrow f$ is bijective; range $f = \text{domain } f^{-1}$.
- Graph of f^{-1} : reflect f in $y = x$.
- Restricted branch: the root's sign must match the branch.
- Increasing f : $f(x) = f^{-1}(x) \Leftrightarrow f(x) = x$.

STANDARD RESULTS

$(x-2)/(x-3) \rightarrow (3x-2)/(x-1) \cdot e^x - e^{-x} \rightarrow \ln((x + \sqrt{x^2 + 4})/2)$

TRAPS

- ✗ **Choosing the wrong sign of the root** Match the branch: $x \geq 2$ needs $+\sqrt{}$.
- ✗ **Keeping the old domains when inverting pieces** The inverse lives on the ranges of the pieces.
- ✗ **Reading f^{-1} as $1/f$** f^{-1} undoes f ; $1/f$ is the reciprocal.

6 One-One, Many-One, Onto and Into Functions

- One-one: equal outputs force equal inputs; horizontal line test.
- Strictly monotonic $\Rightarrow$ one-one; restrict the domain to fix many-one.
- Onto: range = codomain; always read the codomain.

STANDARD RESULTS

$x^2/(x^2 + 1)$: range $[0, 1)$ • $(x^2 - 4)/(x^2 + 1)$: many-one, into, range $[-4, 1)$.

TRAPS

- ✗ **Saying a function is onto without its codomain** x^2 is into on $\mathbb{R} \rightarrow \mathbb{R}$ but onto on $\mathbb{R} \rightarrow [0, \infty)$.
- ✗ **Confusing the two line tests** Vertical: is it a function? Horizontal: is it one-one?
- ✗ **Including the asymptote value in the range** $x^2/(x^2 + 1)$ never equals 1.

9 Inequalities and the Wavy Curve Method

- $\times$ negative flips the inequality; $a < b > c$ says nothing about a, c .
- Square and invert ranges with the graphs, not the ends.
- Wavy curve: $+$ on the far right; odd power changes sign, even bounces; exclude denominator zeros.

STANDARD RESULTS

$-3 < x \leq 4$: $x^2 \in [0, 16]$, $1/x \in (-\infty, -1/3) \cup [1/4, \infty)$

TRAPS

- ✗ **Squaring the ends of an interval containing 0** x^2 starts at 0 there.
- ✗ **Cross-multiplying by an expression of unknown sign** Bring everything to one side and use the sign scheme.
- ✗ **Including zeros of the denominator** The expression is undefined there.

7 Bijections, Counting Functions and Composites

- Bijective = one-one + onto.
- f, g one-one (onto) $\Rightarrow \text{gof one-one (onto)}$; converse forces only f one-one and g onto.
- f onto, g not one-one $\Rightarrow \text{gof not one-one}$; f not onto, g one-one $\Rightarrow \text{gof not onto}$.

STANDARD RESULTS

$5 \rightarrow 3$ onto: 150 • $4 \rightarrow 3$ onto: 36 • $4 \rightarrow 2$ onto: 14

10 Domain and Range of Algebraic Functions

- Complete the square, bound the inside, then apply $\sqrt{}$ or $1/t$ with their graphs.
- Discriminant method: y in range $\Leftrightarrow$ real x ; check the ends and the case where x^2 vanishes.
- $f(x) \geq 0 \forall x \Leftrightarrow a > 0, D \leq 0$.

STANDARD RESULTS

$(x^2 - x + 1)/(x^2 + x + 1)$: $[1/3, 3]$ • $1/(x^2 - x - 1)$: $(-\infty, -4/5] \cup (0, \infty)$

TRAPS

- ✗ **Not checking the ends from $D \geq 0$** Test whether each end is reached; $a = 1$ above fails.
- ✗ **Forgetting the x^2 coefficient can vanish** Treat $y = 1$ (or $a = 0$) separately.
- ✗ **Taking $1/t$ over an interval that contains 0 as one piece** Split at $t = 0$.

11 Modulus Function

- $|x|$ = distance from 0; $|x - a|$ = distance from a .
- $y = |x - a|$: V with vertex $(a, 0)$.
- $|f(x)|$: flip the part below the x -axis up.

STANDARD RESULTS

$$-3 \leq x < 1 \Rightarrow |x| \in [0, 3] \cdot x + |x|: \text{range } [0, \infty)$$

TRAPS

- ✗ **Writing $\sqrt{(x^2)} = x$** $\sqrt{(x^2)} = |x|$.
- ✗ **Taking $|\cdot|$ of the interval ends only** If the interval crosses the vertex, the range starts at 0.
- ✗ **Believing $x^2 \neq |x^2|$ (book misprint)** $x^2 = |x|^2 = |x^2|$ for every real x .

12 Trigonometric Functions: Graphs and Ranges

- $\sin, \cos: \mathbb{R} \rightarrow [-1, 1]$, period 2π ; $\tan, \cot$: range $\mathbb{R}$, period π ; $\sec, \csc$: $|y| \geq 1$.
- $\cos, \sec$ even; $\sin, \tan, \cot, \csc$ odd.
- $\sin x = \sin \alpha \Rightarrow n\pi + (-1)^n \alpha$; $\cos x = \cos \alpha \Rightarrow 2n\pi \pm \alpha$;
 $\tan x = \tan \alpha \Rightarrow n\pi + \alpha$.

STANDARD RESULTS

$$a \cos x + b \sin x \in [-\sqrt{(a^2 + b^2)}, \sqrt{(a^2 + b^2)}]$$

TRAPS

- ✗ **Using $|a| + |b|$ for the range of $a \cos x + b \sin x$** It is $\sqrt{(a^2 + b^2)}$.
- ✗ **Giving $\tan x$ a period of 2π** $\tan$ and $\cot$ repeat every π .
- ✗ **Putting values between -1 and 1 in the range of $\sec x$** $|\sec x| \geq 1$.

13 Domain and Range of Trigonometric Expressions

- Name the trig part t , find its interval, apply the outer function's graph.
- $\sqrt{(\sec^2 x)} = |\sec x|$; hidden domain limits make intervals open.
- Count solutions as graph crossings; $|\sin x| \leq 1$ bounds the search.

STANDARD RESULTS

$$\sin^2 x - \sin x + 1: [3/4, 3] \cdot 1/(2 \cos x - 1): (-\infty, -1/3] \cup [1, \infty) \cdot \sin x = x/10: 7$$

TRAPS

- ✗ **Simplifying $\sqrt{(1 + \tan^2 x)}$ to $\sec x$** It is $|\sec x|$, and $\tan x$ must exist.

- ✗ **Plugging only the ends of $t \in [-1, 1]$** Check whether the vertex lies inside.
- ✗ **Counting crossings beyond $|x| \leq k$ for $\sin x = x/k$** $|\sin x| \leq 1$ limits the search.

14 Inverse Trigonometric Functions

- $\sin^{-1}: [-1, 1] \rightarrow [-\pi/2, \pi/2]$; $\cos^{-1}: [-1, 1] \rightarrow [0, \pi]$;
 $\tan^{-1}: \mathbb{R} \rightarrow (-\pi/2, \pi/2)$.
- Odd: $\sin^{-1}, \tan^{-1}, \csc^{-1}$; $\pi - f(x)$: $\cos^{-1}, \cot^{-1}, \sec^{-1}$.
- Pairs add to $\pi/2$; $\tan^{-1}(1/x)$ needs $-\pi$ for $x < 0$.
- $\sin^{-1}(\sin x) = x$ only on $[-\pi/2, \pi/2]$.

TRAPS

- ✗ **Writing $\sin^{-1}(\sin x) = x$ for all x** Only on $[-\pi/2, \pi/2]$.
- ✗ **Using $\tan^{-1}(1/x) = \cot^{-1} x$ for negative x** For $x < 0$ it is $-\pi + \cot^{-1} x$.
- ✗ **Giving $\cos^{-1}(-x) = -\cos^{-1} x$** $\cos^{-1}(-x) = \pi - \cos^{-1} x$.

15 Exponential and Logarithmic Functions

- $a^x: \mathbb{R} \rightarrow (0, \infty)$; $\log_a x: (0, \infty) \rightarrow \mathbb{R}$; mirror images in $y = x$.
- Base > 1 keeps inequalities; $0 < \text{base} < 1$ flips them.
- Variable base: > 0 and $\neq 1$; argument > 0 .

STANDARD RESULTS

$$\log_{10}(10 - 10^x) \text{ is its own inverse.}$$

TRAPS

- ✗ **Not flipping for a base between 0 and 1** $\log$ base $\frac{1}{2}$ is decreasing.
- ✗ **Ignoring base conditions for a variable base** The base must be positive and not 1.
- ✗ **Forgetting the argument must be positive** $\log$ needs argument > 0 .

16 Greatest Integer, Fractional Part, Signum and Max-Min

- $[x]$: largest integer $\leq x$; $[x + n] = [x] + n$.
- $\{x\} = x - [x] \in [0, 1)$, period 1; $\{x\} + \{-x\} = 0$ or 1.
- $\text{sgn } x = |x|/x$ ($x \neq 0$); max/min = upper/lower envelope.
- Equations: solve for $[x]$, then convert to intervals.

STANDARD RESULTS

$$[x]^2 - 5[x] + 6 = 0 \Rightarrow x \in [2, 4)$$

TRAPS

- ✗ **$[-2.3] = -2$** $[x]$ rounds down: $[-2.3] = -3$.
- ✗ **$\{-2.3\} = 0.3$** $\{-2.3\} = -2.3 - (-3) = 0.7$.
- ✗ **Solving for x instead of $[x]$** Solve for $[x]$, then turn each value into an interval.

17 Even and Odd Functions

- Domain symmetric about 0 first.

- Even: y-axis symmetry; odd: origin symmetry; odd at $0 \Rightarrow f(0) = 0$.
- Products like signs; g even $\Rightarrow fog$ even.
- Even extension $f(-x)$, odd extension $-f(-x)$.

TRAPS

- ✗ **Calling a function even or odd on a non-symmetric domain** Then it is neither.
- ✗ **Forgetting $f(x) = 0$ is both** It is the only function that is both on a given domain.
- ✗ **Assuming odd $\times$ odd is odd** Products follow sign rules: odd $\times$ odd is even.

- $\{f(x)\}$ jumps where f crosses integers; $f(\{x\})$ is constant on each $[n, n + 1)$.

TRAPS

- ✗ **Shifting $f(x + k)$ to the right** Inside changes act in reverse: $f(x + k)$ moves left.
- ✗ **Confusing $|f(x)|$ with $f(|x|)$** $|f(x)|$ flips the part below the x-axis; $f(|x|)$ mirrors the right half.
- ✗ **Treating $|y| = f(x)$ as a function** It mirrors the upper part down: not a function.

18 Periodic Functions

- Fundamental period: least positive T with $f(x + T) = f(x)$.
- $f(ax + b)$: $T/|a|$.
- Sums: LCM of fractions = LCM(num)/HCF(den); then test smaller candidates.
- Outer even functions can halve the period: $\cos(\sin x)$ has period π .

STANDARD RESULTS

$|\sin x| + |\cos x|$: $\pi/2$ · $\sin^4 x + \cos^4 x$: $\pi/2$ · $\sin 2x + \cos 3x$: 2π

TRAPS

- ✗ **Taking the LCM as the final answer** Test smaller candidates when there are moduli or even powers.
- ✗ **Letting the shift b change the period of $f(ax + b)$** Only a matters: $T/|a|$.
- ✗ **Assuming every sum of periodic functions is periodic** $\sin x + \{x\}$ is not periodic.

19 Functional Equations

- Substitute $x = y = 0$, $y = -x$, $x = y = 1$, $y = 1/x$.
- $f(x + y) = f(x) + f(y) \rightarrow kx$; $f(xy) = f(x) + f(y) \rightarrow k \log x$;
 $f(x + y) = f(x)f(y) \rightarrow a^x$.
- Swap x and $1/x$ (or $1 - x$) to get a solvable system.

TRAPS

- ✗ **Confusing the additive and exponential families** Sums of values: kx ; products of values: a^x .
- ✗ **Forgetting the zero function in $f(x + y) = f(x)f(y)$** $f(0) = 0$ gives $f \equiv 0$; otherwise $f(0) = 1$.
- ✗ **Substituting without a plan** Start with 0 , $-x$, 1 and $1/x$.

20 Transformation of Graphs

- $f(x) + k$ up; $f(x + k)$ left; $kf(x)$ vertical stretch; $f(kx)$ horizontal squeeze.
- $-f(x)$: reflect in x-axis; $f(-x)$: reflect in y-axis.
- $|f(x)|$ flips below up; $f(|x|)$ mirrors right half; $|y| = f(x)$ mirrors upper part down.

Intervals () end excluded · [] end included

Periods sin, cos, sec, cosec 2π · tan, cot π · {x} 1

Principal ranges $\sin^{-1}$: $[-\frac{\pi}{2}, \frac{\pi}{2}]$ $\cos^{-1}$: $[0, \pi]$ $\tan^{-1}$: $(-\frac{\pi}{2}, \frac{\pi}{2})$

1 Relations: Definition, Domain and Range

$$n(A \times B) = pq \text{ when } n(A) = p, n(B) = q$$

$$\text{relations from } A \text{ to } B = 2^{pq}$$

2 Reflexive, Symmetric and Transitive Relations

$$\text{reflexive: } 2^{n^2-n}$$

$$\text{symmetric: } 2^{(n^2+n)/2}$$

$$\text{reflexive and symmetric: } 2^{(n^2-n)/2}$$

3 Equivalence Relations and Bell Numbers

$$B_{n+1} = \sum_{k=0}^n {}^nC_k B_k, B_0 = 1$$

$$B_1 = 1, B_2 = 2, B_3 = 5, B_4 = 15$$

4 Functions: Definition, Intervals and Domain

$$(a, b)^c = (-\infty, a] \cup [b, \infty)$$

$$[a, b]^c = (-\infty, a) \cup (b, \infty)$$

- Function: each input exactly one output; vertical line test.

5 Algebra of Functions and Composite Functions

- $D(f \pm g) = D_f \cap D_g$; f/g also drops $g = 0$; $D(\text{gof}) = \{x \in D_f : f(x) \in D_g\}$.

6 One-One, Many-One, Onto and Into Functions

- One-one: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$; onto: range = codomain.

7 Bijections, Counting Functions and Composites

$$\text{functions } A \rightarrow B: m^n$$

$$\text{one-one: } {}^mP_n \ (m \geq n)$$

$$\text{onto: } \sum_{r=0}^m (-1)^r {}^mC_r (m-r)^n$$

$$\text{bijections } (m = n): n!$$

8 Inverse Functions

$$f \circ f^{-1} = I_B, f^{-1} \circ f = I_A$$

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

- f^{-1} exists $\Leftrightarrow$ bijective; graph: mirror in $y = x$.

9 Inequalities and the Wavy Curve Method

$$x^2 \leq a^2 \Leftrightarrow -a \leq x \leq a$$

$$x^2 \geq a^2 \Leftrightarrow x \leq -a \text{ or } x \geq a$$

$$ab > 0: \frac{1}{x} \in [\frac{1}{b}, \frac{1}{a}]$$

$$a < 0 < b: \frac{1}{x} \in (-\infty, \frac{1}{a}] \cup [\frac{1}{b}, \infty)$$

10 Domain and Range of Algebraic Functions

$$f(x) = a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a^2} \right]$$

$$a > 0: \text{range } [-\frac{b^2}{4a}, \infty)$$

$$a < 0: \text{range } (-\infty, -\frac{b^2}{4a}]$$

11 Modulus Function

$$|x| = x \ (x \geq 0), -x \ (x < 0)$$

$$|f(x)| = f(x) \ (f \geq 0), -f(x) \ (f < 0)$$

$$|ab| = |a| |b|$$

$$|\frac{a}{b}| = \frac{|a|}{|b|}$$

$$x^2 = |x|^2 = |x^2|$$

$$|x| = a \ (a > 0) \Rightarrow x = \pm a$$

$$|a \pm b|^2 = a^2 + b^2 \pm 2ab$$

$$(|a| \pm |b|)^2 = a^2 + b^2 \pm 2|a||b|$$

12 Trigonometric Functions: Graphs and Ranges

$$\sin x \geq 0 \Leftrightarrow x \in \bigcup [2n\pi, (2n+1)\pi]$$

$$\cos x \geq 0 \Leftrightarrow x \in \bigcup [2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}]$$

$$a \cos x + b \sin x = \sqrt{a^2 + b^2} \sin(x + \alpha)$$

$$\text{range } [-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$$

13 Domain and Range of Trigonometric Expressions

- $\sqrt{1 + \tan^2 x} = |\sec x|$. $\sin x = x/10$: 7 roots.

14 Inverse Trigonometric Functions

$$\sin^{-1}(-x) = -\sin^{-1} x$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

$$\tan^{-1}(-x) = -\tan^{-1} x$$

$$\cot^{-1}(-x) = \pi - \cot^{-1} x$$

$$\sec^{-1}(-x) = \pi - \sec^{-1} x$$

$$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$$

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, |x| \leq 1$$

$$\sec^{-1} x + \text{cosec}^{-1} x = \frac{\pi}{2}, |x| \geq 1$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, x \in \mathbb{R}$$

$$\sin^{-1} \frac{1}{x} = \text{cosec}^{-1} x, \cos^{-1} \frac{1}{x} = \sec^{-1} x \ (|x| \geq 1)$$

$$\tan^{-1} \frac{1}{x} = \cot^{-1} x \ (x > 0), -\pi + \cot^{-1} x \ (x < 0)$$

15 Exponential and Logarithmic Functions

- Base > 1 keeps, base in $(0, 1)$ flips; variable base $> 0, \neq 1$.

16 Greatest Integer, Fractional Part, Signum and Max-Min

$$x - 1 < [x] \leq x$$

$$[x] \geq n \Leftrightarrow x \geq n$$

$$[x] < n \Leftrightarrow x < n$$

$$\text{sgn } x = 1, 0, -1 \text{ for } x > 0, x = 0, x < 0$$

$$\text{sgn } x = \frac{|x|}{x}, x \neq 0$$

$$\max(a, b) = \frac{a+b+|a-b|}{2}$$

$$\min(a, b) = \frac{a+b-|a-b|}{2}$$

17 Even and Odd Functions

$$f(x) = \frac{f(x)+f(-x)}{2} + \frac{f(x)-f(-x)}{2}$$

- $E \times E = O \times O = E$; $E \times O = O$. Domain must be symmetric.

18 Periodic Functions

$$\text{period of } f(ax + b) = \frac{T}{|a|}$$

- Sums: LCM, then test smaller ($|\sin x| + |\cos x|$: $\pi/2$).

19 Functional Equations

- Additive $\rightarrow kx$; $f(xy) = f + f \rightarrow k \log x$; $f(x + y) = f \cdot f \rightarrow a^x$.

20 Transformation of Graphs

- Outside: as written; inside: reversed ($f(x + k)$ left).