

Thermodynamics Laws & Specific Heats of Gases

$R \ 8.314 \text{ J/mol} \cdot \text{K}$

$1 \text{ atm } 1.013 \times 10^5 \text{ Pa}$

$\text{Signs } Q > 0 \text{ heat in} \cdot W > 0 \text{ work by gas}$

$\gamma \text{ mono } 5/3 \cdot \text{di } 7/5$

1 Thermal Equilibrium and the Zeroth Law

- Zeroth law: $A \sim C$ and $B \sim C \Rightarrow A \sim B$. Equal T, not equal energy.

2 First Law of Thermodynamics

$dQ = dU + dW$

$\Delta Q = \Delta U + \Delta W$

$dQ = nC dT$

$Q = nC(T_2 - T_1)$

$dW = P dV$

$W = \int_{V_1}^{V_2} P dV$

$dU = \frac{f}{2} nR dT$

$\Delta U = \frac{f}{2} nR(T_2 - T_1)$

- Signs: $Q > 0$ heat in, $W > 0$ work by the gas. U depends only on T.

3 Indicator Diagrams, State Variables and Cycles

$W = \int_{V_1}^{V_2} f(V) dV$

$\Delta U_{12} = Q - W \text{ (any path)}$

$\Delta U_{\text{cycle}} = 0$

$Q_{\text{net}} = W_{\text{net}} = \text{loop area}$

- Point = state; area under path = W . Cycle: $\Delta U = 0$, $W_{\text{net}} = \text{loop area}$ (+ clockwise, - anticlockwise).

4 Molar Specific Heats: C_v , C_p and γ

$C_v = \frac{f}{2} R$

$C_p = C_v + R = \frac{f+2}{2} R$

$\gamma = \frac{C_p}{C_v} = \frac{f+2}{f}$

$\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} = \frac{n_1 + n_2}{\gamma_{\text{eq}} - 1}$

- Mono: $C_v = 3R/2$, $\gamma = 5/3$ · di: $C_v = 5R/2$, $\gamma = 7/5$ · $C_v = R/(\gamma - 1)$.

5 Isochoric, Isobaric and Isothermal Processes

$\text{isochoric: } W = 0$

$Q = \Delta U = nC_v \Delta T = \frac{nR\Delta T}{\gamma - 1}$

$\text{isobaric: } W = nR(T_2 - T_1)$

$Q = nC_p \Delta T$

$\Delta U = nC_v \Delta T \text{ (any process)}$

$\text{isothermal: } PV = \text{const}$

$W = nRT \ln \frac{V_2}{V_1} = nRT \ln \frac{P_1}{P_2}$

$Q = W$

- Always $\Delta U = nC_v \Delta T$. Isothermal: $W = nRT \ln(V_2/V_1)$.

6 Adiabatic Process

$\text{adiabatic: } PV^\gamma = \text{const}$

$TV^{\gamma-1} = \text{const}$

$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{nR(T_1 - T_2)}{\gamma - 1}$

- Adiabatic slope = $\gamma \times$ isotherm slope. $B_{\text{iso}} = P$, $B_{\text{adi}} = \gamma P$.

7 Free Expansion of a Gas

$Q = 0, W = 0, \Delta U = 0$

$P_1 V_1 = P_2 V_2$

- Free expansion: $Q = W = \Delta U = 0$; T unchanged (ideal gas).

8 Polytropic Process

$PV^n = \text{const}$

$C = \frac{R}{\gamma - 1} + \frac{R}{1 - n}$

$\frac{dP}{dV} = -n \frac{P}{V}$

- $n = 0, 1, \gamma, \infty \rightarrow$ isobaric, isothermal, adiabatic, isochoric.

9 Reversible Processes, Heat Engines and Refrigerators

$W = Q_1 - Q_2$

$\eta = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}$

$\text{COP} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$

- Engine: $W = Q_1 - Q_2$, $\eta = 1 - Q_2/Q_1$. Fridge: $\text{COP} = Q_2/W$.

10 Second Law and the Carnot Cycle

$\eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1}$

- Kelvin–Planck: no 100% engine. Clausius: no cold $\rightarrow$ hot without work.